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Adaptive Green Time Allocation Method for Traffic Congestion Based on Cell Transmission Model and Genetic Algorithm

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Abstract

Traffic congestion is defined as a physical phenomenon relating to the manner in which vehicles impede each other's progression as demand for limited road space approaches full capacity. This makes trip time longer and increasing queuing. Also it causes serious problems for the day to day lives of people, massive financial and man-hour loss, environment pollution, some diseases etc. In Sri Lanka, traffic congestion in a given area occurs for many reasons. The main reason is that the demand of road does not match to road capacity. In Sri Lanka, although an increase of 10% per year road demand is expected, it can increase road capacity by around 2% to 3% per year. Other important reasons are the existing traffic control system and traffic intersections. Traffic control systems play a central role of traffic management in Sri Lankan cities. Existing traffic light system mainly controls the traffic light change in constant cycle time. But road conditions in a given area vary day by day. If the traffic control system does not deal with these variations, then traffic control system will create bottlenecks and delays. Therefore, the control of traffic requires adequate adjustments to these variations. In this research, we focus on studying and applying cell transmission model to dynamic traffic signal controlling procedure. Basic cell transmission model is used to model vehicular traffic flow and to estimate the total delay of vehicles in a given region. To find an optimal signal timing plan, the Genetic Algorithm is used. The proposed model is applied with certain assumptions to find an optimal time plan to a signalized intersection in main Kandy - Colombo road which has heavy traffic congestion in the morning hours in weekdays. A section of this region is selected to minimize the total delay and to find an optimal dynamic time plan for the signal lights using the actual data collected in this region. The results are *compared with the existing pre-timed signal time plan and the corresponding total delay. It is observed that the proposed dynamic signal timing plan will reduce average delay by 6.2675% and it can be proposed as an alternative for the existing system.*

Key words: *Cell transmission model, Genetic algorithm, Dynamic signal light plan.*

1. Introduction

Traffic congestion is defined as a physical phenomenon relating to the manner in which vehicles impede each other's progression as demand for limited road space approaches full capacity. This makes trip time longer and increasing queuing. Also it causes serious problems for the day to day lives of people, massive financial and man-hour loss, environment pollution, some diseases etc.

In Sri Lanka, traffic congestion in a given area occurs for many reasons. The main reason is that the demand of road does not match to road capacity. In Sri Lanka, although an increase of 10% per year road demand is expected, it can increase road capacity by around 2% to 3% per year. Other important reasons are the existing traffic control system and traffic intersections.

Traffic control systems play a central role of traffic management in Sri Lankan cities. Existing traffic light mainly controls the traffic light change in constant cycle time. However, road conditions in a given area vary day by day. If the traffic control system does not deal with these variations then traffic control system will create bottleneck and delays. Therefore, the control of traffic requires adequate adjustments to these variations.

A traffic intersection defined by a location in which the sharing of road right of way by two or more traffic streams is required. To accomplish this sharing, intersection control is used. The traffic signal operates by allocating green time to the intersection approaches by predetermined way. Basically there are 3 methods to allocate this green time:

(a) Pre-timed or fixed: Provide fixed amount of green time to each approaches during a cycle. The green time duration is fixed for each interval for some period of time.

(b) Actuated: provide minimal length of green time to each approach during a cycle. The length of green time depends on the vehicle arrivals to the each approach and this is determined by using detective devices.

(c) Adaptive: The traffic signals provide green time to each intersection approaches based on awaited arrivals for a cycle. Generally, as arrivals change from cycle to cycle, the length of green time provided to each approaches is changed.

In Sri Lanka almost all traffic signal light use fixed or actuated green time allocation methods. Observing traffic congestion at an intersection it can clearly identify the time allocation method and this provides a major solution for traffic congestion. Therefore developing adaptive green time allocation method for traffic signal control system would result in an effective solution to congestion.

Kiribathgoda city situated in the Western Province is one of the busiest towns in Sri Lanka. Many people pass this city every day because one entrance to the Colombo city 2lies through this city. Therefore main road within this city suffers a heavy traffic jam during school and office hours. According to our observations, basically this traffic jam begins around 7.00 a.m. to 7.15 a.m. in every week day. There is approximately 1.8 km from Gala junction to Kiribathgoda - Sapugaskanda signalized intersection on the main Kandy- Colombo road and in any week day morning it will approximately take 15 to 30 minutes to travel this distance.

Preliminary observations on this problem reveal that major reasons for this congestion are the presence of many yellow cross sections, unsignalized intersections within this region and major bus stops near the unsignalized intersections and the inappropriate time plan at signalized intersections using pre-timed or fixed time method.

2. Problem Statement

In this study we develop an adaptive green time allocation method for intersections which are controlled by traffic signal lights by using Cell Transmission Model (CTM) and Genetic Algorithm (GA) on current road condition of each approaches in order to minimize the delay in a given road system. To validate the model, we use the road section from Gala Junction to Kiribathgoda -Sapugaskanda road signalized intersection and data for the analysis are collected. The data collection was done by using video cameras from 7.15 a.m. to 7.45 a.m. in weekdays.

3. Objective of the Research

The objective in this research is to study and apply traffic operating system that operates using real time situation and adjusts the green time of a cycle according to the current situation in order to minimize the total delay in a given road intersection.

4. Theoretical Framework

4.1 Cell Transmission Model

The cell transmission model originally proposed by Daganzo(1994) as a discrete approximation to the LWR model[7]. In CTM each link in a traffic network is divided into one or more interconnected cells. The cell length is not arbitrarily chosen. The cell length is taken as distance traveled by vehicles in one simulation time step at the free flow speed.

Each pair of cell may be interconnected with one connector only as in figure 3.7, and multiple connectors between an upstream cell and downstream cell are not permitted. Connectors have no physical dimension and it represents the interface between two consecutive cells. Also we assume that vehicles can advance at most one cell under free flow condition.

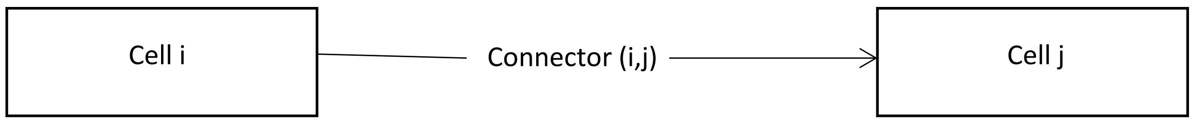

Figure 3.7: cells and connector representation in CTM

4.1.1 Network topology for CTM

Given any network, let

$\mathcal{C}$ = set of all cells

$\mathcal{E}$ = set of connectors

Each cell j in the set $\mathcal{C}$ may connect to a set of cells. Let

$\Gamma^{-1}(j)$ = set of predecessor cells which connect to cell j

$\Gamma(j)$ = set of successor cells which connect to cell j

Each cell/connector is classified into one of five distinct types:

Cell $j \in \mathcal{C}$ is defined as

Ordinary if	$\lvert\Gamma^{-1}(j)\rvert = 1$	and	$\lvert\Gamma(j)\rvert = 1$ (see figure 3.8. (a))
Merging if	$\lvert\Gamma^{-1}(j)\rvert > 1$	and	$\lvert\Gamma(j)\rvert = 1$ (see figure 3.8. (b))
Diverging if	$\lvert\Gamma^{-1}(j)\rvert = 1$	and	$\lvert\Gamma(j)\rvert > 1$ (see figure 3.8. (c))
Source if	$\lvert\Gamma^{-1}(j)\rvert = 0$	and	$\lvert\Gamma(j)\rvert = 1$ (see figure 3.8. (d))
Sink if	$\lvert\Gamma^{-1}(j)\rvert = 1$	and	$\lvert\Gamma(j)\rvert = 0$ (see figure 3.8. (e))

Figure 3.8 shows 5 types of cell organization in graphically.

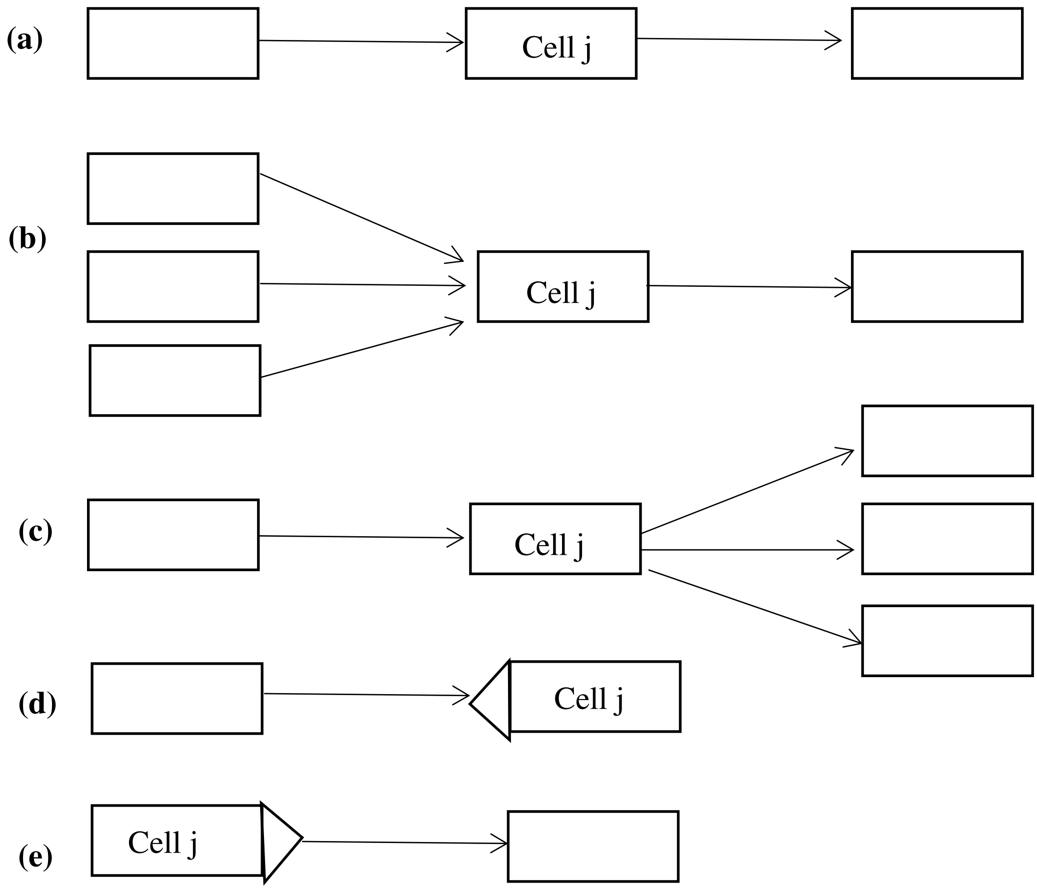

Figure 1: Network topology in CTM

4.1.2 Basic assumptions in CTM

The original CTM can use only for freeway traffic flow. After introducing some variations to the model we can use this model for any road segment. Also assume that there is a piecewise linear relationship between flow and density in a cell. This relationship between flow and density is depicted in figure 2 and is expressed by

$$q = min\{VK_j, Q_{max}, w(K_j - k)\}$$, where

V = The free flow speed of highway segment
K_j = Jam density
Q_{max} = The maximum flow rate (capacity of cell)
w = Speed of backward moving waves
k = Current density

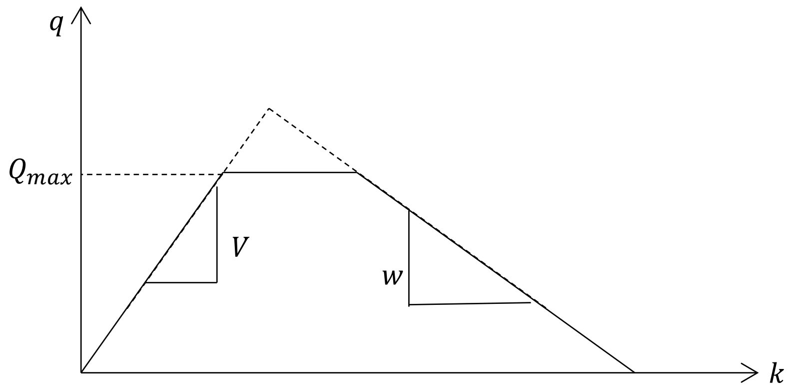

Figure 2: Fundamental diagram for CTM

4.1.3 Flow advancing equations in CTM

4.1.3.1 Ordinary cells and connectors

The flow advancing equation for an ordinary connector (i, j) is

$$y_{ij}(t, t + \Delta t) = \min\{\, n_i(t), \min\left[Q_i \,, Q_j\right] \,, \delta_j\left(N_j - n_j(t)\right) \} \qquad \forall (i, j) \,\epsilon\, \mathcal{C}$$

Where

$y_{ij}(t, t + \Delta t)$ = Number of vehicles advancing from cell i to cell j in one simulation time step Δt at time t,

$\mathcal{C}$ = Set of all ordinary cells,

$n_i(t), n_j(t)$ = The occupancy of cell i and j at time t,

N_j = Maximum occupancy of cell j,

$Q_i \,, Q_j$ = the flow capacity of cells i and j during Δt.

δ_j = 1 if $n_i(t) \leq Q_j$

= $\frac{w_j}{v_j}$ otherwise

4.1.3.2 Merging cells and connectors

For each merging cell j in the set of merging cells $\mathcal{C}_\mu$, the inflow on merging connectors are determined by the following simple linear program.

$$MAX \sum_{\forall i \in \Gamma^{-1}} y_{ij}(t)$$

subject to:

$$y_{ij}(t, t + \Delta t) \leq \min\{n_i(t), Q_i, Q_j, \rho_j \delta_j (N_j - n_j)\} \qquad \forall i \in \Gamma^{-1}(j)$$

Here ρ_i be the merging priorities. This is a quantitative rule that assigns right of way to merging vehicles from road with different hierarchies. In here we use,

$$\rho_i \leq 1 \qquad \forall i \in \Gamma^{-1}(j)$$

$$\sum_{\forall i \in \Gamma^{-1}(j)} \rho_i = 1$$

4.1.3.3 Diverging cells and connectors

For each diverging cell j of set of diverging cells $\mathcal{C}_{\mathcal{D}}$ the outflow of diverging cells are determined by using the following simple linear program.

$$Max\ \sum_{\forall j \in \Gamma(i)} y_{ij}$$

Subject to:

$$y_{ij}(t, t+\Delta t) \leq \min\{\gamma_j Q_j\ , \gamma_j \delta_j (N_j - n_j(t))\}\ ,\ \forall j \in \Gamma(i)$$

$$\sum_{\forall j \in \Gamma(i)} y_{ij} \leq \min\{n_i(t), Q_i\}$$

$$\gamma_j \leq 1, \qquad \forall j \in \Gamma(i)$$

$$\sum_{\forall j \in \Gamma(i)} \gamma_j = 1$$

where γ_j is the turning ratio from cell i to cell j. It can also be defined as the probability that a vehicle turn from cell i to cell j.

4.1.3.4 Source and sink

The sink for all existing traffic, there should be infinite size (($N = \infty$) and a suitable possibly time varying, capacity. Input flows can be modeled by a cell pair.

Source cell (numbered 00) with an infinite number of discharge ($n_{00}(0) = \infty$) that discharge into an empty gate cell 00 of infinite size. The inflow capacity of the gate cell is set to the desired link input flow time interval $t + 1$.

4.1.3.5 The flow conservation equation

After determining connectors of flows of the cells using linear programs and equations described in sections 3.3.3.1 to 3.3.3.3, we can apply the following conservation equation to determine the cell occupancy in each cell at the next time step, $t + \Delta t$.

$$n_j(t + \Delta t) = n_j(t) + \sum_{\forall i \in \Gamma^{-1}(j)} y_{ij}(t) - \sum_{\forall k \in \Gamma(j)} y_{jk}(t) \qquad \forall j \in \mathcal{C}$$

where $\mathcal{C} = set\ of\ all\ cells.$

4.1.3.6 Delay on a link

Within CTM, **approach delay** (in time steps) at cell i ($d_i(t)$)can be estimated by subtracting a cell's outflow($y_i(t)$) from its occupancy($n_i(t)$) for each time step.

$$d_i(t) = n_i(t) - y_i(t)$$

Total delay in the network can be found by summing $d_i(t)$ over $t\ and\ i$. Thus
Total delay=$\sum_t \sum_i d_i(t)$

The average delay (per vehicle) can be calculated by dividing total delay by the number of affected vehicles.
Average delay=Total delay/number of affected vehicles.

Initially, for empty network, the number of affected vehicles is equal to the number of vehicles that enter the system during simulation period. In CTM, at beginning the network is not empty and the number of affected vehicles equal to the number of vehicles on the link at the beginning of the analysis period plus the number of vehicles that enter the link during simulation the period.

4.2 Genetic Algorithm

Genetic algorithm (GA) is an adaptive heuristic search algorithm based on the evolutionary ideas of natural selection and genetics.

GA begins with a population of randomly generated members called chromosomes. Each chromosome has genes and firstly GA requests that each individuals in the population have its fitness evaluated. This evaluation done in the fitness function. This function returns fitness value of each individual.

After GA completely evaluating the population, GA operates on these members to form a new population. All new members have characteristics of the old population, but all new members are different from old ones and are fit than old ones. Therefore all members should be evaluated. As this process will do with evaluation criteria and GA operators until individuals reach to required fitness value.

4.3 Represent CTM as a Fitness Function

Using the theory and equations which are discussed, we can model any given road section using CTM. In CTM the given road section is divided into homogeneous road segments and using conservation equations and advancement equations we can predict vehicle occupancy in each cell at every simulation time step. As mentioned above, it is easy to calculate the average delay per vehicle using CTM. We consider the junction given in figure 3. Assume that there are only two phases at the junction, either green (red) for main corridor (Cross Street) traffic vice versa. The allowed directions to travel of vehicles are denoted using arrows. The ring diagram for this scenario is given in table 1.

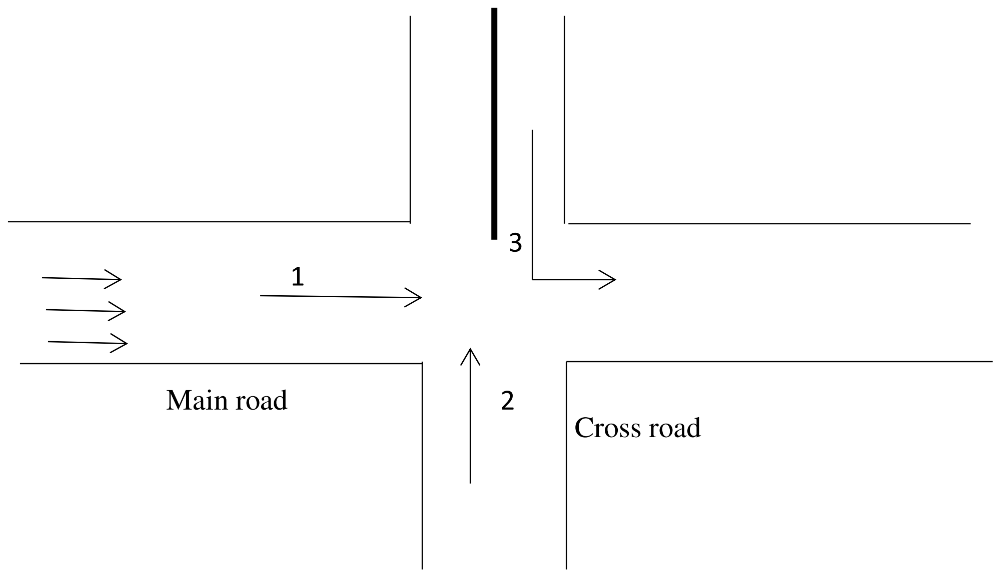

Figure 3: Intersection with two phases

Table 1: Ring Diagram for junction given in figure 3

	Main	Cross
Ring 1	1 →	3 ↳
Ring 2		2 ↑
	Phase 1	Phase 2

Assume that the cycle length of traffic signal is fixed (let this be τ seconds) and
$\tau > simulation\ time\ step\ \Delta t$.

Also assume that we need to find best signal time plan for reducing the total delay within next T second time interval. Suppose $T > \tau$.

Number of cycles in (0,T) time interval=$\left[\frac{T}{\tau}\right] + 1$

Consequently, we need to allocate green time of this cycle to each phase. Let the fraction of time which allocate to phase1 in cycles 1 ,2,3,…,n be t_1 , t_2 , …, t_n respectively, where $0 < t_i \leq 1$, for $i = 1,2,3, \dots, n$.

Then,
Green time allocation for phase 1 in cycle 1 =$t_1\tau$= red time of phase 2 in cycle 1
Green time allocation for phase 1 in cycle 2 =$t_2\tau$= red time of phase 2 in cycle 2
.
.
.
Green time allocation for phase 1 in cycle (n-1) =$t_{n-1}\tau$= red time of phase 2 in cycle n-1 .
Green time allocation for phase 1 in cycle n =$t_n\tau$= red time of phase 2 in cycle n

If we assume that road conditions are ideal, then given initial cell occupancies and given origin flows, the total delay of vehicles in this region totally depends only on these green time allocation ratios. Using CTM, simulate τ second, we can get average delay per vehicle and Total delay corresponding to different time allocation ratio values t_i within next $(0, T)$ time interval. Therefore, given road section, CTM can be used as fitness function and $t_i's$ are the input (or decision variables) and the average delay per vehicle as output. Then using Genetic algorithm, it can be find the near optimal time allocation of signal cycle in order to minimize the total delay per vehicle in the system for next time interval$(0, T)$ can be found.

5. Model Implementation

5.1 Scenario overview

We have chosen road section from Gala junction to Kandy- Sapugaskanda signalized intersection. In this scenario we only consider vehicles flow towards Colombo. In this road section there are many unsignalized intersections and yellow cross sections. However, we only consider one unsignalized junction located at near YMBA bus stop. Plan view of this site is given in figure 5. According to our observations, average speed of a vehicle in the main road is $36\ kmh^{-1}$. Assume that this speed is the same movements within simulation time period. Flow directions of these road sections are also shown in the figure 5.

Figure 4: Street map from Gala junction to Kandy-Sapugaskanda signalized intersection

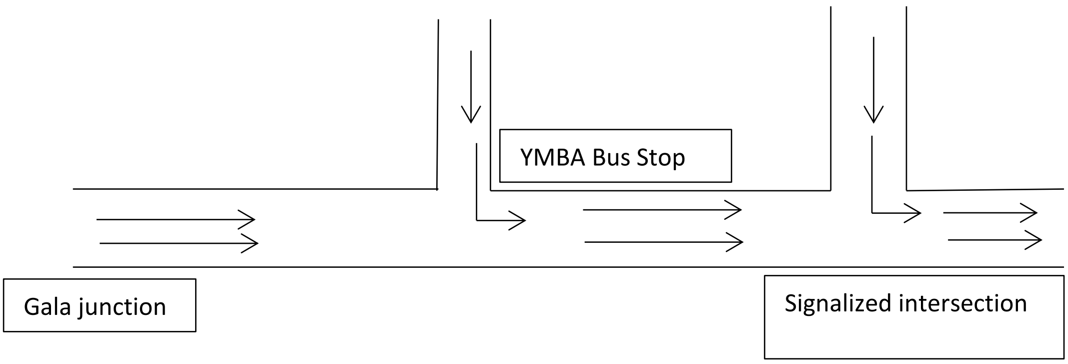

Figure 5: Site view of the road section

5.2 Parameters

To apply CTM-GA based traffic signal timing plan model to this site, it is required to estimate model parameters. This estimation is done by using collected data within the time period 7.15 a.m to 7.45 a.m in week days.

5.2.1 Physical road features

From Google map and by site investigation the length of required road sections and number of lanes can be easily recognized. All of this information are given in figure 6.

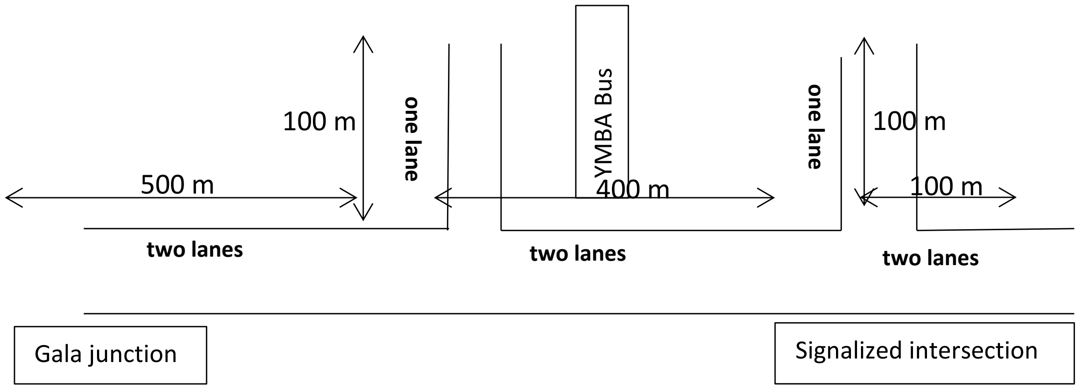

Figure 6: number of lanes and lengths of required section

5.2.2 Flow characteristic

5.2.2.1 Free flow speed

As mentioned in Section 5.1 free flow speed is taken as $36\ kmh^{-1}$ for all roads in the considered region. Also assume that the speed is constant. Simulation time step is taken as 10 seconds. Then cell length of each road section is$100m$. Cell representation of the road section given in the figure 7. Also note that there are three origin cells in this cell representation and one sink.

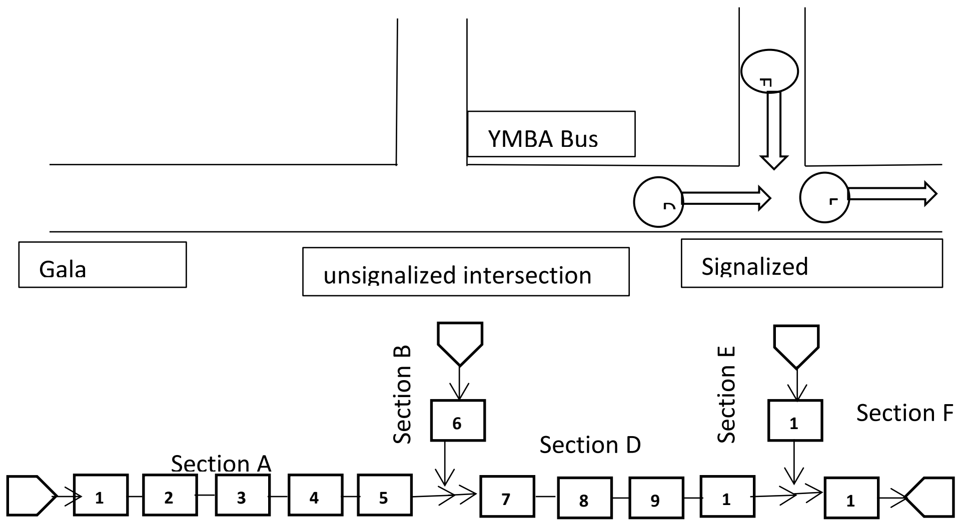

Figure 7: Cell representation of the road section

5.2.2.2 Shock wave Speed

Shock wave speed describes the speed for a standing queue to dissipate and assume it is equal to $36\ kmh^{-1}$ (i.e. assume that the basic Cell transmission model that shock wave speed=free flow speed).

5.2.2.3 Jam Density

Weighted average of a vehicle length is taken as the average length of vehicle. Vehicle lengths taken from Google and each length are weighted by the frequency of each vehicle category in the collected data set. Then the Jam density (k_j),

$$k_j = \frac{1}{average\ length\ of\ a\ vehicle(in\ km)} \times number\ of\ lanes$$

5.2.2.4 Cell Capacity

Capacity of each cell calculated by using the following equation,

$$Cell\ Capacity = Jam\ density \times cell\ length \times number\ of\ lanes$$

Capacity of cells in each road section are shown in the table 2.

Table 2: Cell capacity of each road section

Section	Cell capacity(vehicles)
A	64.81
B	32.4
D	64.81
E	32.4
F	64.81

5.2.2.4 Saturation flow rate

The saturation flow rate is the highest flow of vehicles that can pass through an intersection under the existing traffic conditions in 100% green time. The data collected near the signalized intersection are used to calculate saturation flow rate. The estimated value of saturations flow rate is 2085.4963 vehicles per hour per lane.

Maximum flow that can advance in each time step= $Q_{max} \times duration\ of\ time\ step$

Maximum flow that can advance in each time step in each section are shown in the table 3.

Table 3: Maximum flow that can advance in each time step

Section	Maximum flow (vehicles per time step)
A	11.58
B	5.79
D	11.58
E	5.79
F	11.58

5.2.2.6 Merging ratios of unsignalized intersection

The merge ratio is the flow ratio of two directional flows from one directional flow.

Since the intersection near YMBA bus stop is not a signalized intersection, we need to estimate merge ratios. The estimated ratios are 0.8 and 0.2 for main road and cross road respectively.

5.3 Signal timing plan

- Cycle time

In this implementation, we take cycle time as 2 minutes and assumed that each signal phase has only green and red phase. The 10 minutes time period have 5 cycles.

Since simulation done for 10 minutes time period, CTM and GA based signal time plan described in chapter 4 gives the following time plan for next 10 minutes time interval in order to minimize the expected total delay. Simulation was performed using the MATLAB. Results obtained from this program are necessarily based on the initial cell occupancy and rates of flow coming from origins (Source cells). In this analysis, there are 3 Source cells and three different set of flow rates from each origin are used to this calculation (table 4). In each case, we use the same initial cell occupancies (table 5). Results are summarized in the table 6.

Case 1: Used day1 flow of origin data
Case 2: Used day2 flow of origin data
Case 3: Used day3 flow of origin data

Table 4: Flow from each origin

Section	Flow of origin(vehicles per time step)		
	Day 2	Day 3	Day 4
A	9.34	6.11	11.68
B	1.72	3.06	1.85
E	4.39	7.34	5.86

Table 5: Initial number of vehicles in each cell

Initial Occupancy	32	32	32	32	32	16	32	32	32	32	32	30
Cell	1	2	3	4	5	6	7	8	9	10	11	12

Table 6: Results

Cycle	Case 1		Case 2		Case 3	
	Green time for Approach D	Green time for approach E	Green time for Approach D	Green time for Approach E	Green time for Approach D	Green time for Approach E
1	90.352007	29.647993	86.460883	33.539117	96.000000	24.000000
2	95.086088	24.913912	95.577651	24.422349	96.000000	24.000000
3	88.693628	31.306372	90.420587	29.579413	95.636877	24.363123
4	90.507210	29.492790	89.628547	30.371453	95.292845	24.707155
5	95.039985	24.960015	66.190238	53.809762	96.000000	24.000000
Average Expected delay(in minutes)	6.530487		7.232746		6.378850	

To compare the existing fixed time plan with this allocation, we used CTM for fixed time plan to estimate average delay per vehicle. According to the observation, Green time allocations of each approach are same in existing traffic light system. Results are shown in table 7.

Table 7: Average delay per vehicle for dynamic and fixed signal timing plan

Case	Proposed dynamic plan (average delay per vehicle)(minutes)	Constant signal timing plan (average delay per vehicle)(minutes)	Expected delay reduction (as percentage of delay at constant signal timing plan)
Case 1	6.530487	6.708034	2.647%
Case 2	7.232746	6.874606	-5.210%
Case 3	6.378850	7.078767	9.888%

Table 7 represents the dynamic time allocation plan for next 10 minutes time period. Note that this allocation depends on initial cell occupancy and flow rate which come from origins.

According to the table 7, in case 2 fixed time plan gives the minimum delay. This could be due to the fact that genetic algorithm gives near optimal solution. Increasing the maximum number of generations we can handle this error. Considering all values of the table, we can conclude that signal allocation using proposed method is better than the existing fixed timing plan.

6. DISCUSSION AND CONCLUSIONS

In this research, we develop a method based on Cell Transmission Model and the Genetic Algorithm to control signalized intersection dynamically. Using the traffic data in Kiribathgoda town, an optimal traffic signal dynamic plan is introduced to a signalized intersection on the main Colombo – Kandy road. The proposed signal timing plan will reduce average delay by 6.2675%.

The suggested time schedule is valid for the next 10 minute time interval and according to the traffic flows from each origin and the initial cell occupancy this allocation will change. Therefore we need to update these values using video cameras and vehicle recognition software and accurate results can be obtained by applying this procedure.

Because of the practical difficulties, we used only one signalized intersection in Kiribathgoda town for this implementation. Generally the developed model can be applied to any number of signalized intersections in any region. However, when the number of signalized intersections or simulation duration increases, the number of decision variables and time to find optimum allocation will increase. Further in this implementation we only consider two phases in each split of the ring diagram; red phase and green phase. The other important amber color phase will be included to the proposed model is our future work.

When applying Cell Transmission Model we are compelled to assume that there is a homogeneous traffic flow and road conditions. However the driving behavior, weather condition, movement of pedestrians, number of yellow cross section always disturb the homogeneity. Therefore we need to take in to account these factors to improve the results of the proposed method.

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